MATH 54 - LIST OF CHEAT SHEET - TYPOS/SUGGESTIONS

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Here are a couple of typos on the old cheat sheet I found so far! 1) and 2) are absolutely crucial, so make sure you correct your cheat sheets.

Update: There's another typo (see (10)), and I added two suggestions (11) and (12). I won't update the cheat sheets again, because those are just very small typos and easy to spot, but make sure to note them!

Update: See (14) for a suggestion on how to solve $\mathbf{x}'' = A\mathbf{x}$ in case Grunbaum asks for it!

(1) **IMPORTANT** In Grunbaum's mass-spring system for

$$N = 3$$

and on the H.O. handout,

$$\mathbf{v_3} = \begin{bmatrix} \frac{\sqrt{2}}{2} \\ -1 \\ \frac{\sqrt{2}}{2} \end{bmatrix}$$

I put $-\frac{\sqrt{2}}{2}$ instead of $\frac{\sqrt{2}}{2}$.

(2) Also, for the coupled mass-spring system, the general case, the matrix A is wrong, it's supposed to be:

$$A = \begin{bmatrix} -2 & 1 & 0 & \cdots & 0 & \\ 1 & -2 & 1 & 0 & \cdots & 0 \\ 0 & 1 & -2 & 1 & 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & -2 & 1 \\ 0 & 0 & 0 & 0 & 1 & -2 \end{bmatrix}$$

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(3) For the Fourier sine series, it's supposed to be:

$$b_m = \frac{2}{T}\int_0^T f(x)\sin\left(\frac{m\pi x}{T}\right)$$

Btw, the formulas for a_0 are **CORRECT**. It looks different from the formula from the book, but that's because the book divides by 2 at the *end*, i.e. the book considers $\frac{a_0}{2}$. Here I'm dividing by 2 at the beginning so that you won't have to think about this anymore when you're calculating a_m.

(4) For the heat equation, step 4, it's supposed to be

$$T(t) = \widetilde{A_m}e^{\lambda t}$$

.

(5) For the quadratic forms-section, it's supposed to be

$$\mathbf{y} = P^T\mathbf{x}$$

(6) The definition of the Wronskian of 2 function is wrong:

$$\widetilde{W}(t) = \begin{bmatrix} f(t) & g(t) \\ f'(t) & g'(t) \end{bmatrix}$$

(7) For 'orthogonality', it should be

$$\mathbf{y} = c_1\mathbf{u_1} + \cdots + c_n\mathbf{u_n}$$

(8) **IMPORTANT** For the generalized eigenvectors, it should be $(A - \lambda I)$, not $(A + \lambda I)$

(9) On some cheat sheets, it says if Case 2 *doesn't* work, start with $m = 0$. That should be $m = 1$.

(10) For undetermined coefficients, I put

$$y_p(t) = t^s\left(A_m t^m + \cdots + A_1 t + 1\right)e^{rt}$$

That 1 is supposed to be an A_0 (just a constant). Similarly for the other equation with $\sin$ and $\cos$.

(11) This is not a typo, but if you ever deal with an equation of the form

$$(D+3)(D^2+4)[y]=0$$

remember that for the aux. equation, you replace D by r, so you get

$$(r+3)(r^2+4)=0$$

(12) Again, not a typo, but for the first two cheat sheets, I didn't put convergence of Fourier series because Grunbaum said it won't be on the exam. But if you have space, just for the heck of it, if you have space, write down that the Fourier series goes to $f(x)$ if f is continuous at x, and to the average of the jumps if f has a jump at x, and to the average of the endpoints at the endpoints!

(13) Abel's lemma: See handout (under the mock final)

(14) In the unlikely event that Prof. Grunbaum asks for the general solution to your system $\mathbf{x}'' = A\mathbf{x}$, where A is your harmonic oscillator matrix (mass-spring system), here's how you do it:

You should get $2N$ linearly independent solutions:

$$\cos(\omega t)\mathbf{v} \qquad \text{and} \qquad \sin(\omega t)\mathbf{v}$$

where ω are your proper frequencies (without the i and without the minus, so if you have $\pm\sqrt{3}i$, you would do $\omega = \sqrt{3}$) and $\mathbf{v}$ are your proper modes.

Then take linear combinations of those.

Finally, if he asks you to plug in $\mathbf{x}(0)$ and $\mathbf{x}'(0)$, then solve for the $2N$ constants you found! (you will have to differentiate $\mathbf{x}$ for the $\mathbf{x}'(0)$ part, so you indeed get $2N$ equations in $2N$ unknowns!)